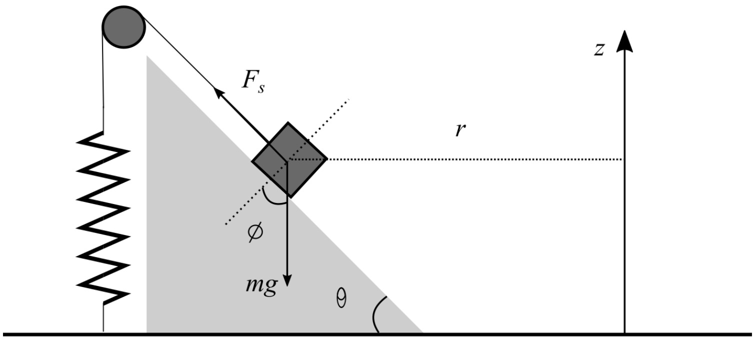
These are sample questions. Actual exam will be different than these problems, but the style and level will be similar.

1. (30 points) Consider a mass m which can slide frictionlessly on the inclined plane. The mass is attached to a vertical spring with spring constant k through an unstretchable string passing over a frictionless pulley. The whole setup is fixed to a table which can rotate frictionlessly about z-axis, as shown in the figure. We only focus on the motion of the mass m.



   (a) (2 points) Show that angle $\phi = \theta$.

   (b) (8 points) First consider that the table is at rest. Due to the gravity, the spring is stretched by amount d. The force F_s is the tension in the string due to the stretched spring. What is the magnitude of F_s? Find out the stretched length d of the spring in equilibrium.

   (c) (4 points) Let's now consider that the table is rotating about z-axis with constant angular velocity ω. The rotation will cause a reaction force F_r, opposite to the centripetal force, to act on the mass. Find out the magnitude of F_r in terms of ω.

   (d) (8 points) Due to the rotation, the stretched length of the spring will be readjusted. Calculate the new stretched length d' in the presence of F_r.

   (e) (4 points) How d' compares to d? Will the spring be stretched more or less?

   (f) (4 points) How much angular velocity is required to bring the spring back to its un-stretched length?

2. (30 points) If the angular momentum of a so-called spin-1 particle is measured in a particular direction (conventionally referred to as the z direction), the measurement results obtained are $\hbar$, 0, and $-\hbar$. The postulates of quantum mechanics then dictate that $\hat{S}_z$, the operator corresponding to the angular momentum in the z direction, satisfies $\hat{S}_z|+\hbar\rangle = +\hbar|+\hbar\rangle$, $\hat{S}_z|0\hbar\rangle = 0\hbar|0\hbar\rangle$ and $\hat{S}_z|-\hbar\rangle = -\hbar|-\hbar\rangle$, where $|+\hbar\rangle$, $|0\hbar\rangle$, and $|-\hbar\rangle$ are the eigenstates of $\hat{S}_z$ corresponding to eigenvalues $+\hbar$, $0\hbar$, and $-\hbar$ respectively. The states $|+\hbar\rangle$, $|0\hbar\rangle$, and $|-\hbar\rangle$ form an orthonormal basis (i-e. $\langle i|j\rangle = \delta_{i,j}$). In terms of the states $|+\hbar\rangle$, $|0\hbar\rangle$, and $|-\hbar\rangle$, the operator $\hat{S}_z$ can be written as

$$\hat{S}_z = +\hbar|+\hbar\rangle\langle+\hbar| + 0\hbar|0\hbar\rangle\langle 0\hbar| - \hbar|-\hbar\rangle\langle -\hbar| \tag{1}$$

The corresponding operators for other two components of spin are given as

$$\hat{S}_x = \frac{\hbar}{\sqrt{2}} \left(|+\hbar\rangle\langle 0\hbar| + |0\hbar\rangle\langle +\hbar| + |0\hbar\rangle\langle -\hbar| + |-\hbar\rangle\langle 0\hbar| \right) \quad (2)$$

$$\hat{S}_y = \frac{\hbar}{\sqrt{2}} \left(-i|+\hbar\rangle\langle 0\hbar| + i|0\hbar\rangle\langle +\hbar| - i|0\hbar\rangle\langle -\hbar| + i|-\hbar\rangle\langle 0\hbar| \right). \quad (3)$$

(a) (6 points) Write the operators $\hat{S}_z$, $\hat{S}_x$, and $\hat{S}_y$ as matrices expressed in the basis $|+\hbar\rangle$, $|0\hbar\rangle$, and $|-\hbar\rangle$. Recall that given an operator $\hat{X}$, and basis vectors $|a_1\rangle$, $|a_2\rangle$,..., $|a_N\rangle$, the matrix elements are given by $X_{ij} = \langle a_i|\hat{X}|a_j\rangle$.

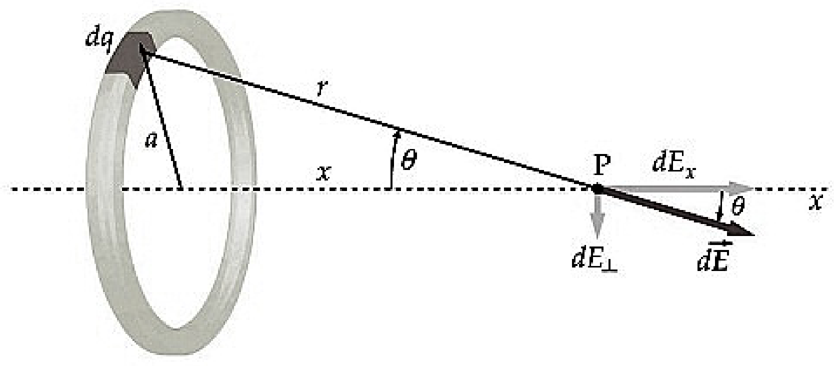
(b) (6 points) Suppose that initially the state of the atom is $|+\hbar\rangle$. If now the angular momentum in the x direction is measured, what are the possible measurement results? You can make a physical argument without any detailed calculations if you so desire.

(c) (6 points) Given these possible measurement results, if the initial state is $|+\hbar\rangle$, and the angular momentum in the x direction is measured, what are the probabilities for obtaining different measurement results? Recall that to find the probabilities, you will need to find the eigenstates of the relevant operator.

(d) (6 points) Starting from the initial state $|\psi(0)\rangle = |+\hbar\rangle$, we now apply a magnetic field in the x direction. The Hamiltonian in such a case is given by $\hat{H} = -\Omega\hat{S}_x$, where Ω is a constant. Express the state $|\psi(t)\rangle$ in terms of $|+\hbar\rangle$, $|0\hbar\rangle$, and $|-\hbar\rangle$. Remember that the time evolution operator is given by $\hat{U}(t) = e^{-i\hat{H}t/\hbar}$

(e) (6 points) Given the state $|\psi(t)\rangle$, if a measurement of the angular momentum in the z direction is performed, what are the possible measurement results, and what are their corresponding probabilities?

3. (30 points) Consider a uniformly charged ring of radius a characterised by the linear charge density $\lambda = dq/ds$ (where ds is the infinitesimal arc length along the ring) placed perpendicular to the x-axis with centre at $(0, 0, 0)$ as shown in the figure. You have to calculate the total electric field $\vec{E}$ at point $P(x, 0, 0)$. Let's proceed as following:



(a) (2 points) What is the total charge q on the ring in terms of the linear charge density λ?

(b) (4 points) What is the electric field $d\vec{E}$ at point P along radial direction $\hat{r}$ due to the infinitesimal charge dq on the ring, as shown in the figure?

(c) (6 points) Calculate the electric field components dE_x and $dE_\perp$ along and perpendicular to the x-axis, respectively. Obtain the expressions in terms of dq, a and x.

(d) (3 points) What is the total electric field component $E_\perp$ due to the total charge on the ring?

(e) (3 points) Calculate total $E_x(x)$ in terms of λ, a and x.

(f) (3 points) Find out $E_x(x)$ in the limits $|x| << a$ and $|x| >> a$.

(g) (3 points) Find out the position x_o for which $E_x(x)$ is maximum. How $E_x(x_o)$ varies with the radius a?

(h) (6 points) Using the above analysis, Sketch schematically the $E_x(x)$ for radius a and $2a$ along $+x$-axis. What do you expect when $a \to 0$ and $a \to \infty$?